

Exploring Algebra and Geometry Concepts with GeoGebra

Throughout their K–12 education, students will spend a considerable amount of time developing mathematics competency. By the time they reach middle school and high school, the focus will be on becoming proficient in algebra and geometry. As a means to motivate and help students explore these topics, we recommend a free, multiplatform software application called GeoGebra (www.GeoGebra.org). GeoGebra provides the opportunity to explore a wide variety of algebraic and geometric concepts through hands-on manipulation of graphs, tables, formulas, and shapes. It is also a convenient and easy way to generate graphics and visuals for presentations, test questions, and homework problems. The software is available for Mac, Windows, and Linux operating systems, and a Web applet version is available that works within your Internet browser.

We use GeoGebra in our classrooms to introduce the “big idea” of linear and

exponential growth patterns. Using the software, we generate a side-by-side comparison of graphs, slopes, and growth factors. We generate each curve separately and paste all into a PowerPoint slide presentation (see **fig. 1**) to introduce the concept of linear and nonlinear (exponential) equations and spur a discussion of similarities and differences.

Directed individual exploration using GeoGebra can reinforce students’

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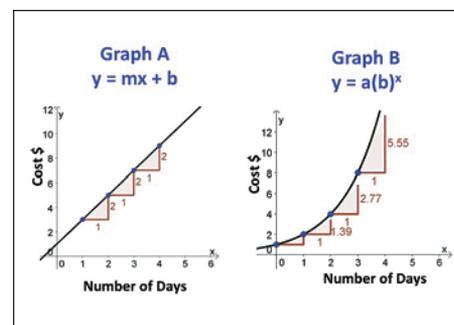


Fig. 1 Using the slope tool in GeoGebra, students can compare nonlinear growth with linear growth.



Fig. 2 The options on the GeoGebra tool bar are easy to understand.

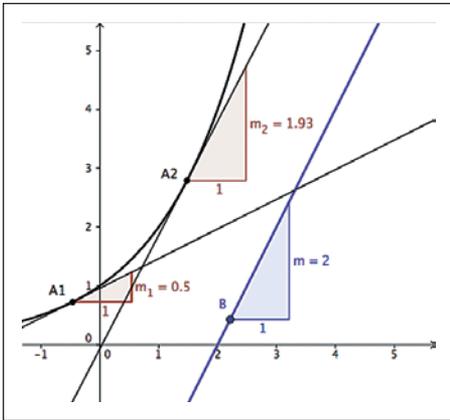


Fig. 3 Points A and B can be dragged along the curve, showing that the slope is constant for the linear function but varies for the exponential function.

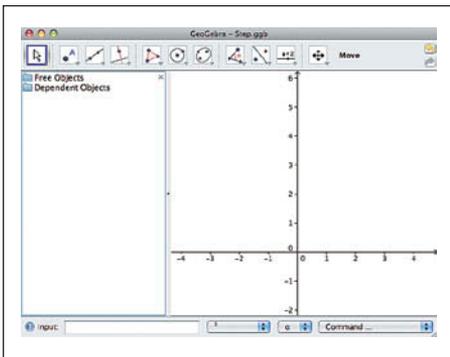


Fig. 4 At the input bar, key in the function $2x$.

understanding of these concepts. The graphical user interface is intuitive, and students will master the functionality quickly. Pull-down menus are indicated by icons and descriptions for categories such as Move, Points, Lines, Perpendicular/Parallel, Polygons, Circles, Ellipses, Angle/Slope Measurement, and Reflection (see **fig. 2**).

In particular, GeoGebra allows students to graph a variety of equations, both linear and nonlinear. Students can enter and graph multiple equations (see **fig. 3**) and add points and tangent lines to any curve. In addition, the software has a built-in slope indicator that displays rise over run and that can be used at a point of tangency. Students can use a mouse to drag the point (and corresponding slope) anywhere on the curve,

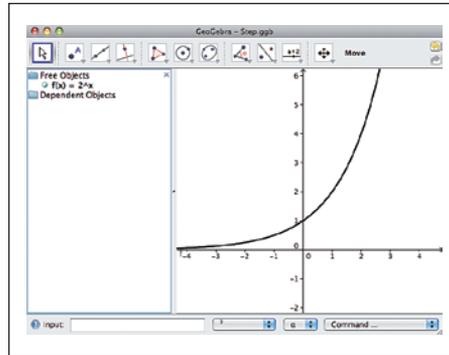


Fig. 5 Pressing Enter will graph the function. GeoGebra chooses the graphing window, but right-clicking on the axes will allow users to set the window manually.

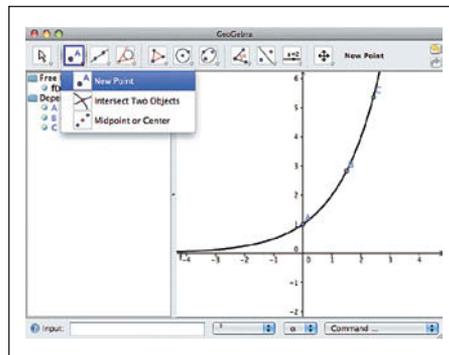


Fig. 6 Points can be added to the curve by using the New Point option on the tool bar.

allowing students to observe how the slope changes as the point slides along the curve. We used this feature to display the changing slope for the exponential curve as well as the constant slope for the linear curve.

Using GeoGebra, students generate individual curves by first entering the appropriate function into the Input Bar. The up arrow is used to indicate exponentiation. Pressing the Enter key will generate the function (see **figs. 4** and **5**).

After graphing the curve, select New Point from the tool bar and place several points on the curve by clicking the function at the desired location (see **fig. 6**). Tangent lines can then be added. To obtain the tangent option, click on the small arrow in the lower-right corner of the Perpendicular Line option on the tool bar. The drop-down menu will have

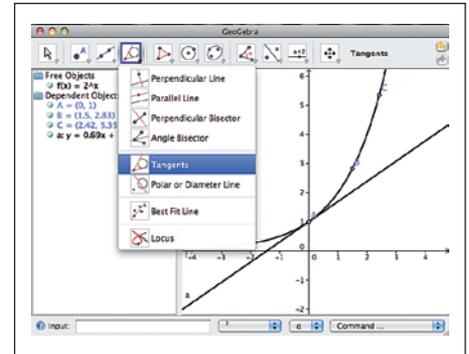


Fig. 7 The drop-down menu accessed through the Perpendicular Lines option allows students to draw tangents to the curve; they can then compare slopes.

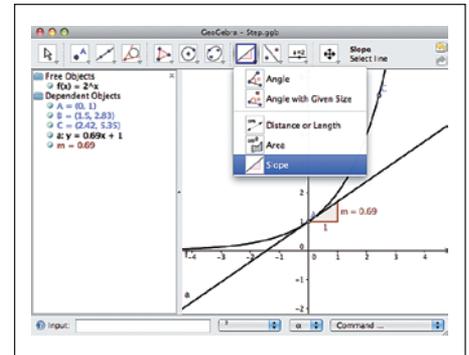


Fig. 8 The slope graphic can be added using the drop-down menu accessible via the Angle icon on the tool bar.

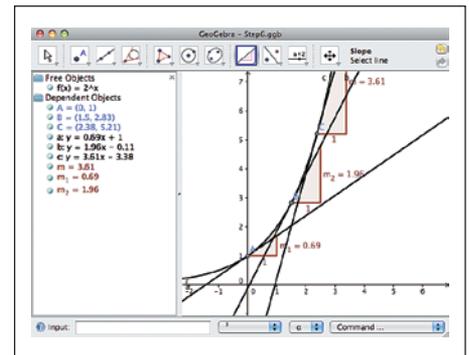


Fig. 9 Repeated use of the slope graphic indicates that slope on the exponential curve varies from one point to another.

the Tangent option (see **fig. 7**). After selecting this option, select the appropriate point and function. The tangent line will be generated automatically.

To generate the slope graphic, access the drop-down menu below Angle on the tool bar, just as was done to access Tangent. Now select Slope and then the tangent line just constructed (see **fig. 8**). This process can be repeated any desired number of times (see **fig. 9**).

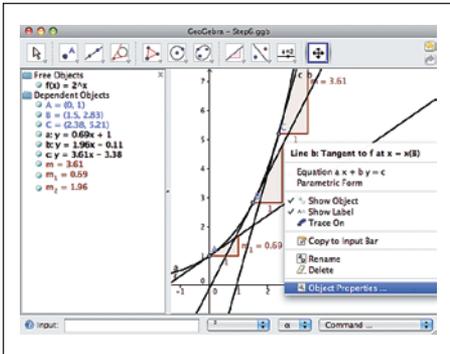


Fig. 10 A right-click on the tangent line produces a dialogue box with the Object Properties option.

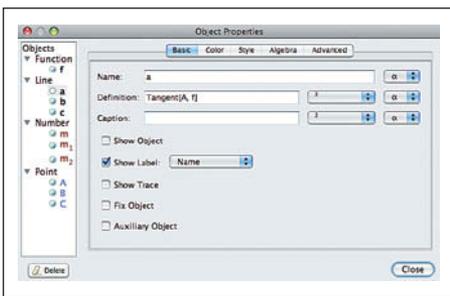


Fig. 11 Note the options available in the Properties menu. Hiding the tangent line might be accomplished more efficiently by deselecting Show Object in the dialogue box in **figure 10**.

To clean up the screen a bit, it might be a good idea to hide the tangent lines, leaving only the slope indicators. To hide a line, right-click on it, thus activating a new menu (see **fig. 10**). Select Object Properties to open still another menu. In the Properties submenu, deselect Show Object to hide the tangent line (see **fig. 11**). Repeating this process for the other tangent lines will leave a much cleaner viewing window. The Properties submenu can also be used to change the color or style of the graph of a function.

GeoGebra files can easily be pasted into other applications, such as MS Word or PowerPoint. To do so, select File, Export, and then Graphics View to Clipboard. The file then becomes available to paste into any other application (see **fig. 12**). Note that the area to the left of the graph, called the Algebra View, contains coordinate information, among other items of interest. Points are labeled with uppercase letters, lines are labeled with lowercase letters, and slopes are labeled with the letter m (see **fig. 13**).

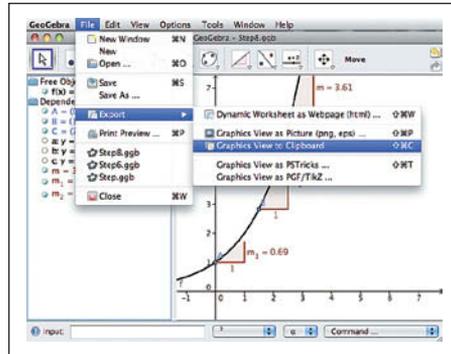


Fig. 12 GeoGebra graphics can be imported to other programs.

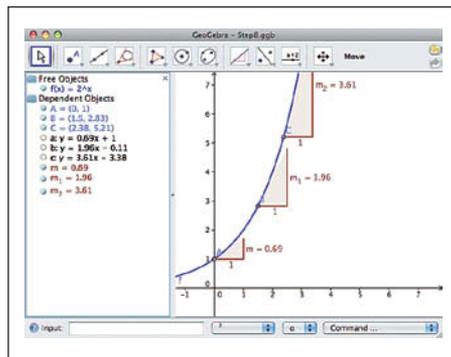


Fig. 13 The Algebra View, the pane to the left of the graph itself, contains a color-coded key to what appears in the Graphics View.

Other activities that students can explore include geometric transformations—such as shifts, stretches, and reflections—for functions such as square root, absolute value, inverse, and polynomial. **Figure 14** demonstrates how the parent function (indicated by the solid black line) can be shifted (indicated by the dashed green line) or reflected (indicated by the dashed red line) progressively. GeoGebra allows for the option of coloring and labeling the lines for further clarification. We use this option to teach students how the equation parameters are related to the movement of the function and how the movement can easily be determined without plotting numerous points.

The examples given here are just a few of the useful algebraic features included in GeoGebra. The possibilities are endless, and the software supports the exploration of both algebraic and geometric concepts. The GeoGebra Web site has a user forum with useful technical assistance and user questions and answers. The Web also contains a plethora of user-developed applets that

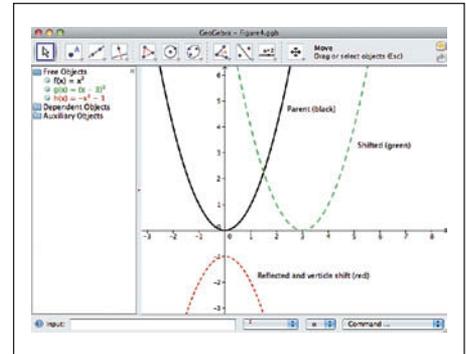


Fig. 14 GeoGebra can also be used to explore transformations.

graphically demonstrate various algebraic and geometric ideas.

We use GeoGebra primarily to present and demonstrate key mathematics concepts but also to develop graphics for visuals and materials for student use. Learning to use this application is straightforward; little instructional and set-up time is needed. The software is free, and the computer requirements are minimal, so students can easily download a copy for home use.

We encourage mathematics educators to investigate the uses of GeoGebra in their classrooms.



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